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## Dyon mass bounds from electric dipole moments

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### Abstract

Dyon loops give a contribution to the matrix element for light-by-light scattering that violates parity and time-reversal symmetry. This effect induces an electric dipole moment for the electron, of order  $M^{-2}$ , where  $M$  is the dyon mass. The current limit on the electric dipole moment of the electron yields the lower mass bound  $M > \mathcal{O}(1)$  TeV.

Very precise measurements achieved during the last decade have opened up for a new approach in elementary particle physics. According to this, evidence of new particles can be extracted from indirect measurements of their virtual contribution to processes at energies which are too low for direct production. For

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example, the top quark mass as predicted from precision electroweak data [1] agrees to within 10% with direct experimental measurements [2].

This approach has recently been applied [3] for the estimation of possible virtual monopole contributions to observables at energies below the monopole mass. Taking into account the violation of parity (and time-reversal symmetry) in a theory with monopoles [4], the emergence of an electric dipole moment was first pointed out by Purcell and Ramsey [5]. More recently, the effect due to monopole loop contributions has been discussed [6].

From our point of view the estimation of the paper [6] needs reexamination. In the present paper we consider first the  $P$  and  $T$  violating contribution of the dyon loop to the matrix element for light-by-light scattering. Then the contribution of this subdiagram to the electric dipole moment of the electron is estimated. The experimental bound on the electron dipole moment leads to a non-trivial bound on the dyon mass.

As was shown in [7], the effective Lagrangian density of QED coupled to dyons contains in the one-loop approximation the fourth-order terms

$$\Delta L \approx \frac{e^4}{360\pi^2 m^4} [(\mathbf{H}^2 - \mathbf{E}^2)^2 + 7(\mathbf{H}\mathbf{E})^2] + \frac{Qg(Q^2 - g^2)}{60\pi^2 M^4} (\mathbf{H}\mathbf{E})(\mathbf{H}^2 - \mathbf{E}^2), \quad (1)$$

where the first term is the familiar Euler-Heisenberg term, with  $e$  and  $m$  the charge and mass of the electron, whereas the second term is a  $P$  and  $T$  non-invariant dyon loop contribution, with  $M$  the dyon mass and  $Q$  and  $g$  its electric and magnetic charges, respectively<sup>2</sup>. This expression yields the matrix element for low-energy photon-photon scattering. In order to determine it, we substitute into (1) the expansion

$$F_{\mu\nu}(x) = \frac{i}{(2\pi)^4} \int d^4q (q_\mu A_\nu - q_\nu A_\mu) e^{iqx}. \quad (2)$$

Corresponding to the second term of (1) we find

$$\begin{aligned} & \frac{Qg(Q^2 - g^2)}{480\pi^2 M^4} \int d^4x \varepsilon_{\mu\nu\rho\sigma} F^{\mu\nu} F^{\rho\sigma} F_{\alpha\beta} F^{\alpha\beta} \\ &= \frac{1}{(2\pi)^{12}} \int d^4q_1 d^4q_2 d^4q_3 d^4q_4 \delta(q_1 + q_2 + q_3 + q_4) A_\mu(q_1) A_\nu(q_2) A_\rho(q_3) A_\sigma(q_4) \widetilde{M}^{\mu\nu\rho\sigma}, \end{aligned} \quad (3)$$

where

$$\widetilde{M}^{\mu\nu\rho\sigma} = \widetilde{M}^{\mu\nu\rho\sigma}(q_1, q_2, q_3, q_4) = \frac{Qg(Q^2 - g^2)}{60\pi^2 M^4} \varepsilon_{\alpha\beta}{}^{\mu\nu} q_1^\alpha q_2^\beta [q_4^\rho q_3^\sigma - g^{\rho\sigma}(q_3 q_4)] . \quad (4)$$

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<sup>2</sup>The Dirac charge quantization condition connects the electric charge of the electron and the magnetic charge of a dyon:  $eg \sim 1$  whereas the dyon electric charge  $Q$  is not quantized.

Symmetrizing this pseudotensor one obtains the  $P$  and  $T$  violating part of the matrix element for light-by-light scattering. With all momenta flowing inwards,  $k_1 + k_2 + k_3 + k_4 = 0$ , the matrix element takes the form

$$\begin{aligned}
M'_{\mu\nu\rho\sigma} &= \frac{1}{6} [\widetilde{M}_{\mu\nu\rho\sigma}(k_1, k_2, k_3, k_4) + \widetilde{M}_{\mu\rho\nu\sigma}(k_1, k_3, k_2, k_4) + \widetilde{M}_{\mu\sigma\nu\rho}(k_1, k_4, k_2, k_3) \\
&\quad + \widetilde{M}_{\nu\rho\mu\sigma}(k_2, k_3, k_1, k_4) + \widetilde{M}_{\nu\sigma\mu\rho}(k_2, k_4, k_1, k_3) + \widetilde{M}_{\rho\sigma\mu\nu}(k_3, k_4, k_1, k_2)] \\
&= \frac{Qg(Q^2 - g^2)}{60\pi^2 M^4} \left( \varepsilon_{\alpha\beta}^{\mu\nu} k_1^\alpha k_2^\beta k_3^\sigma k_4^\rho + \varepsilon_{\alpha\beta}^{\mu\rho} k_1^\alpha k_2^\sigma k_3^\beta k_4^\nu + \varepsilon_{\alpha\beta}^{\mu\sigma} k_1^\alpha k_2^\rho k_3^\nu k_4^\beta \right. \\
&\quad + \varepsilon_{\alpha\beta}^{\nu\rho} k_1^\sigma k_2^\alpha k_3^\beta k_4^\mu + \varepsilon_{\alpha\beta}^{\nu\sigma} k_1^\rho k_2^\alpha k_3^\mu k_4^\beta + \varepsilon_{\alpha\beta}^{\rho\sigma} k_1^\nu k_2^\mu k_3^\alpha k_4^\beta \\
&\quad - \varepsilon_{\alpha\beta}^{\mu\nu} g^{\rho\sigma} (k_3 k_4) k_1^\alpha k_2^\beta - \varepsilon_{\alpha\beta}^{\mu\rho} g^{\nu\sigma} (k_2 k_4) k_1^\alpha k_3^\beta - \varepsilon_{\alpha\beta}^{\mu\sigma} g^{\rho\nu} (k_2 k_3) k_1^\alpha k_4^\beta \\
&\quad \left. - \varepsilon_{\alpha\beta}^{\nu\rho} g^{\mu\sigma} (k_1 k_4) k_2^\alpha k_3^\beta - \varepsilon_{\alpha\beta}^{\nu\sigma} g^{\mu\rho} (k_1 k_3) k_2^\alpha k_4^\beta - \varepsilon_{\alpha\beta}^{\rho\sigma} g^{\mu\nu} (k_1 k_2) k_3^\alpha k_4^\beta \right). \tag{5}
\end{aligned}$$

Since the interaction contains an  $\varepsilon$  tensor, the coupling between two of the photons is different from that involving the other two, and the familiar pairwise equivalence of the six terms does not hold. The matrix element satisfies gauge invariance (with respect to any of the four photons),

$$k_1^\mu M'_{\mu\nu\rho\sigma}(k_1, k_2, k_3, k_4) = 0, \quad \text{etc.} \tag{6}$$

We note that the above contribution to the matrix element is proportional to the fourth power of the inverse dyon mass,  $M'_{\mu\nu\rho\sigma} \propto M^{-4}$ . However, this result is only valid at low energies, where the photon momenta are small compared to  $M$ , being obtained from an effective, non-renormalizable theory.

The contribution of this matrix element breaks the  $P$  and  $T$  invariance of ordinary electrodynamics. Thus, among the sixth-order radiative corrections to the electron-photon vertex there are terms containing this photon-photon scattering subdiagram with a dyon loop contribution (see Fig.1), that induce an electric dipole moment of the electron<sup>3</sup>.

Indeed, one can write the contribution of this diagram to the electron-photon vertex as<sup>4</sup>:

$$\begin{aligned}
\Lambda_\mu(p', p) &= \frac{e^2}{(2\pi)^8} \int d^4 k_1 d^4 k_3 \frac{1}{k_1^2 + i\epsilon} \frac{1}{k_2^2 + i\epsilon} \frac{1}{k_3^2 + i\epsilon} M_{\alpha\beta\gamma\mu}(k_1, k_2, k_3, k) \\
&\quad \times \gamma^\alpha \frac{\not{p}' + \not{k}_1 + m}{(p' + k_1)^2 - m^2 + i\epsilon} \gamma^\beta \frac{\not{p} - \not{k}_3 + m}{(p - k_3)^2 - m^2 + i\epsilon} \gamma^\gamma \tag{7}
\end{aligned}$$

where  $M_{\alpha\beta\gamma\mu}(k_1, k_2, k_3, k)$  is the polarization pseudotensor representing the dyon box diagram contribution to the photon-photon scattering amplitude, the low-energy limit of which is given by the pseudotensor  $M'_{\alpha\beta\gamma\mu}$  of eq. (5).

<sup>3</sup>This has been noted by I.B. Khriplovich [8] — see also a recent paper by Flambaum and Murray [6].

<sup>4</sup>Of course, there are more diagrams.

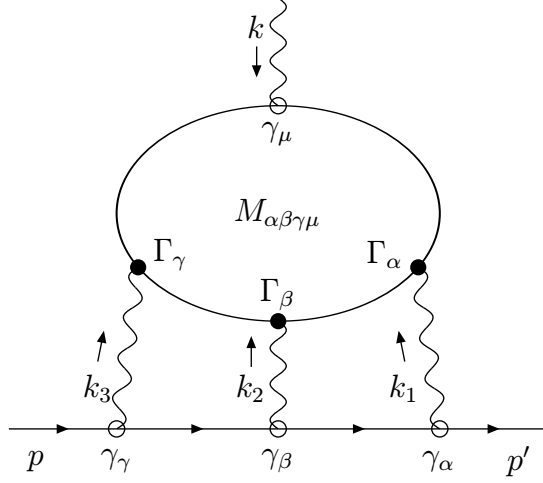


Figure 1: Typical three-loop vertex diagram. The closed line represents a dyon loop.

In order to extract the electric dipole moment from the general expression (7) it is convenient, according to the approach by [9] to exploit the identity

$$M'_{\alpha\beta\gamma\mu}(k_1, k_2, k_3, k) = -k^\nu \frac{\partial}{\partial k^\mu} M_{\alpha\beta\gamma\nu}(k_1, k_2, k_3, k), \quad (8)$$

which can be obtained upon differentiating the gauge invariance condition of the polarization tensor [cf. eq. (6)] with respect to  $k^\mu$ .

Substituting (8) into (7) we can write the  $ee\gamma$  matrix element as

$$\begin{aligned} M_{ee\gamma}(p', p, k) &= e^\mu(k) \bar{u}(p') \Lambda_\mu(p', p) u(p) \\ &= e^\mu(k) k^\nu \bar{u}(p') \Lambda_{\mu\nu}(p', p) u(p) \end{aligned} \quad (9)$$

where  $e^\mu(k)$  is the photon polarization vector and

$$\begin{aligned} \Lambda_{\mu\nu}(p', p) &= -\frac{e^2}{(2\pi)^8} \int d^4 k_1 d^4 k_3 \frac{1}{k_1^2 + i\epsilon} \frac{1}{k_2^2 + i\epsilon} \frac{1}{k_3^2 + i\epsilon} \frac{\partial}{\partial k^\mu} M_{\alpha\beta\gamma\nu}(k_1, k_2, k_3, k) \\ &\quad \times \gamma^\alpha \frac{\not{p}' + \not{k}_1 + m}{(p' + k_1)^2 - m^2 + i\epsilon} \gamma^\beta \frac{\not{p} - \not{k}_3 + m}{(p - k_3)^2 - m^2 + i\epsilon} \gamma^\gamma. \end{aligned} \quad (10)$$

Since the matrix element (9) is already proportional to the external photon momentum  $k$ , one can put  $k = 0$  in  $\Lambda_{\mu\nu}$  after differentiation to obtain the static electric dipole moment.

Then, following [9], we note that due to Lorentz covariance of  $\Lambda_{\mu\nu}$ , it can be written in the form

$$\Lambda_{\mu\nu}(p', p) = \left( \tilde{A} g_{\mu\nu} + \tilde{B} \sigma_{\mu\nu} + \tilde{C} P_\mu \gamma_\nu + \tilde{D} P_\nu \gamma_\mu + \tilde{E} P_\mu P_\nu \right) \gamma_5 + \dots \quad (11)$$

where we have omitted terms that do not violate parity, as well as those proportional to  $k_\mu$ , and where  $\sigma_{\mu\nu} = (\gamma_\mu\gamma_\nu - \gamma_\nu\gamma_\mu)/2$ , and  $P_\mu = p_\mu + p'_\mu$ .

Substituting this expression into the matrix element  $M_{ee\gamma}(p', p, k)$  of eq. (9) one can see that there are two contributions to the  $P$  violating part, arising from the  $\tilde{B}$  and  $\tilde{C}$  terms. In order to project out the dipole moment from (9), one has to compare eq. (11) with the phenomenological expression for the electric dipole moment  $d_e$  [10]:

$$M_{ee\gamma}(p', p, k) = e^\mu(k) k^\nu \bar{u}(p') \frac{d_e}{2m} \gamma_5 \sigma_{\mu\nu} u(p), \quad (12)$$

In the non-relativistic limit it corresponds to the Hamiltonian of interaction  $-(d_e/2m) \boldsymbol{\sigma} \mathbf{E}$ . Thus, multiplying (11) by  $\sigma_{\mu\nu} \gamma_5$  and taking the trace we have:

$$d_e = -\frac{m}{24} \text{Tr} [\sigma_{\mu\nu} \gamma_5 \Lambda^{\mu\nu}]. \quad (13)$$

In order to provide an estimate of the induced electric dipole moment we need to estimate  $\Lambda^{\mu\nu}$ . The first task is to evaluate the polarization pseudotensor  $M_{\alpha\beta\gamma\mu}$  corresponding to the virtual dyon one-loop subdiagram. If we were to substitute for  $M_{\alpha\beta\gamma\mu}$  the low-energy form  $M'_{\alpha\beta\gamma\mu}$  of eq. (5) into eq. (7), we would obtain a quadratically divergent integral.

On the other hand, straightforward application of the Feynman rules in QED with magnetic charge (see, e.g., [11]) would give for the photon-by-photon scattering subdiagram in Fig. 1<sup>5</sup>,

$$M_{\alpha\beta\gamma\mu}(k_1, k_2, k_3, k) = \frac{Qg^3}{2\pi^4} \int d^4q \text{Tr} \left( \Gamma_\alpha \frac{1}{\not{q} + \not{k}_1 - M} \Gamma_\beta \frac{1}{\not{q} - \not{k}_3 - \not{k} - M} \right. \\ \left. \times \Gamma_\gamma \frac{1}{\not{q} - \not{k} - M} \Gamma_\mu \frac{1}{\not{q} - M} \right). \quad (14)$$

Here  $\Gamma_\alpha$  represents the magnetic coupling of the photon to the dyon, which we take according to ref. [11] to be

$$\Gamma_\mu = -i\varepsilon_{\mu\nu\rho\sigma} \frac{\gamma^\nu k^\rho n^\sigma}{(n \cdot k)}. \quad (15)$$

The vertex function depends on  $k^\rho$ , the photon momentum entering the vertex, and on  $n^\sigma$ , a unit constant space-like vector corresponding to the Dirac singularity line. It was shown by Zwanziger [12] that although the matrix element depends on  $n$ , the cross section as well as other physical quantities are  $n$  independent.

Calculations using this technique are very complicated and can only be done in a few simple situations [11], for example, in the case of the charge-monopole scattering problem [13]. We will here avoid this approach.

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<sup>5</sup>It should be noted that the expression (5) contains contributions from such loop diagrams with all possible combinations of either three or one magnetic-coupling vertex  $\Gamma_\rho$ .

While the integration over  $q$  in eq. (14) is logarithmically divergent (the magnetic couplings in (14) are dimensionless), after renormalization the sum of such contributions must in the low-energy limit reduce to the form given in eq. (5). We also note that the substitution of (14) into eq. (10) yields a convergent integral. Thus, the following method for evaluating  $\Lambda_{\mu\nu}$  suggests itself. We divide the region of integration into two domains: (i) the momenta  $k_1$  and  $k_3$  are small compared to  $M$ , and (ii) the momenta are of order  $M$  (or larger).

In the first region, the form (5) can be used, but since the integral is quadratically divergent, the integral will be proportional to  $M^2$ . Together with the over-all factor  $M^{-4}$  this will give a contribution  $\propto M^{-2}$ . For large values of the photon momenta, the other form, eq. (14) can be used. This gives a convergent integral, and dimensional arguments determine the scale to be  $M^{-2}$ . It means that

$$|\Lambda_{\mu\nu}| \sim \frac{e^2 Q g (Q^2 - g^2)}{(4\pi^2)^3 M^2}. \quad (16)$$

The numerical coefficient has been estimated as  $1/(4\pi^2)^3$ , one factor  $1/4\pi^2$  from each loop, and the  $1/24$  of eq. (13) is assumed cancelled by a combinatorial factor from the number of diagrams involved. This is of course a very rough estimate.

Now we can estimate the order of magnitude of the electron dipole moment generated by virtual dyons. It is obvious from Eqs. (13) and (16), that in order of magnitude one can write

$$d_e \sim \frac{e^2 Q g (Q^2 - g^2)}{(4\pi^2)^3} \frac{m}{M^2}. \quad (17)$$

This estimate can be used to obtain a new bound on the dyon mass. Indeed, recent experimental progress in the search for an electron electric dipole moment [15] gives a rather strict upper limit:  $d_e < 9 \cdot 10^{-28} e$  cm. If we suppose that  $Q \sim e$ , from (17) one can obtain  $M \geq 2 \cdot 10^6 m \approx 10^3$  GeV. This estimate shows that the dyon mass belongs at least to the electroweak scale.

The above estimate coincides with the bound obtained by De Rújula [3] for monopoles, from an analysis of LEP data, but it is weaker than the result given in [6], where the limit  $M \geq 10^5$  GeV was obtained. The authors of ref. [6] used the hypothesis that a radial magnetic field could be induced due to virtual dyon pairs. In order to estimate the effect, they used the well known formula for the Ueling correction to the electrostatic potential, simply replacing the electron charge and mass with those of the monopole. But the Ueling term is just a correction to the scalar Coulomb potential due to vacuum polarization and cannot itself be considered as a source of radial magnetic field. Indeed, there is only one second order term in the effective Lagrangian that can violate the  $P$  and  $T$  invariance of the theory, namely  $\Delta L' \propto \mathbf{E} \mathbf{H}$ . But in the framework of QED there is no reason to consider such a correction because it is just a total derivative. The reference to the  $\theta$ -term, used in [6] to estimate the electric charge of the dyon, is only relevant

in the context of a non-trivial topology (e.g., in the 't Hooft-Polyakov monopole model) where their limit applies. In this case there are arguments in favour of stronger limits on the monopole (dyon) mass (see, e.g., [17]).

Finally, one should note that the dyon loop diagram considered above can also contribute to the neutron electric dipole moment. The experimental value  $d_n < 1.1 \cdot 10^{-25} e \text{ cm}$  [16] will in the naive quark model with  $m \approx 10 \text{ MeV}$  allow us to obtain a similar estimate of the dyon lower mass bound as obtained for the electron.

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